



GEM CEDAR modeling challenge: ionospheric conductivity

**State of ionospheric conductivity research
and what's coming next (or should)**





Outline

Fundamentals - Current Understanding/Modeling - What's next?

Fundamentals:

What is conductivity and what is needed to calculate it?

Current understanding/modeling:

What is the current state of understanding based on our previous modeling?

What's next:

One new approach and significance



Outline

Fundamentals - Current Understanding/Modeling - What's next?

Fundamentals:

What is conductivity and what is needed to calculate it?

Current understanding/modeling:

What is the current state of understanding based on our previous modeling?

What's next:

A new app

Conclusions:

Different approaches = different pictures

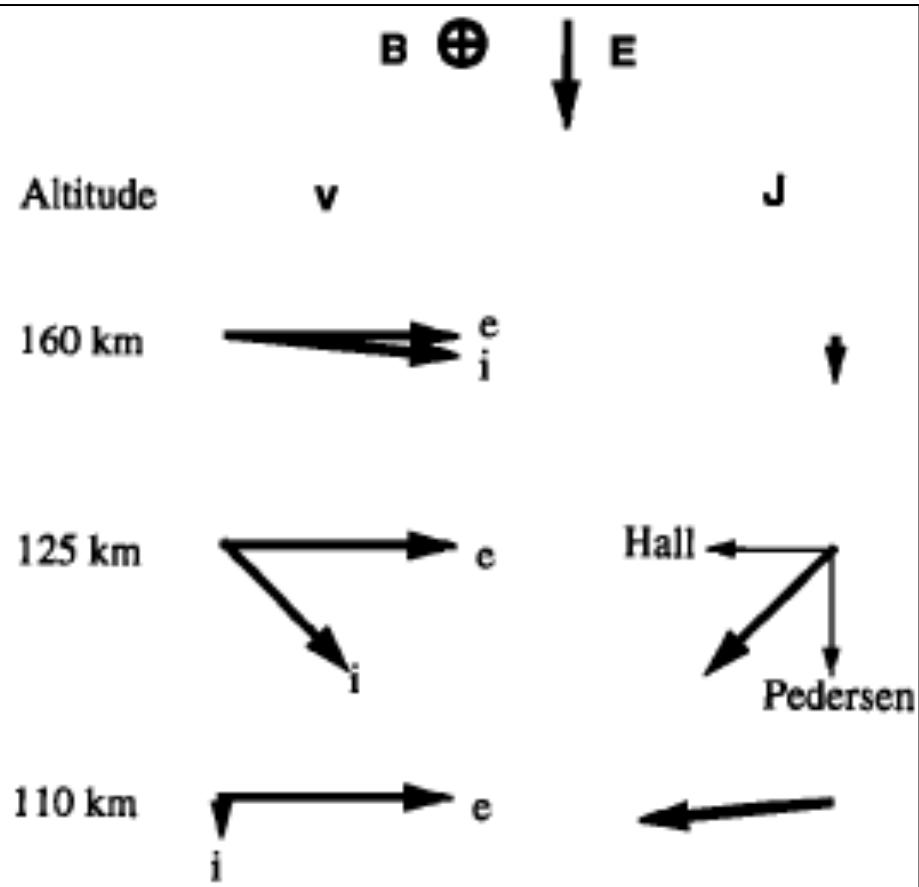
Consistent picture dependent on bringing together diverse data

Creating a capable model that can be implemented in MI models is the goal

Ionospheric Conductivity



Fundamentals - Current Understanding/Modeling - What's next?



$$\mathbf{J} = \tilde{\sigma} \cdot \mathbf{E}$$

$$= \sigma_P \mathbf{E}_\perp + \sigma_H \hat{\mathbf{B}} \times \mathbf{E} + \sigma_\parallel \mathbf{E}_\parallel$$

$$\tilde{\sigma} = \begin{bmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_\parallel \end{bmatrix}$$

$$\int_h \sigma_x dh = \Sigma_x$$

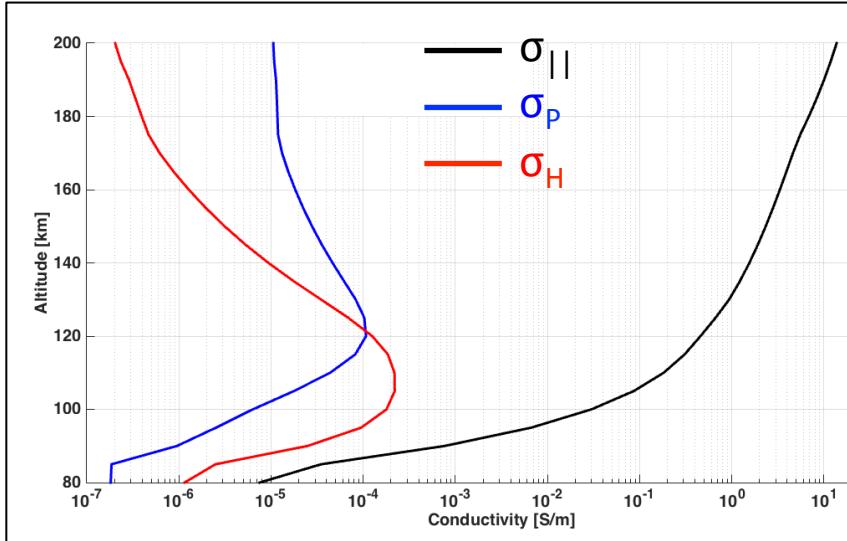
[Richmond and Thayer \[2013\]](#)

Ionospheric Conductivity



Fundamentals - Current Understanding/Modeling - What's next?

Typical profiles



$$\mathbf{J} = \tilde{\sigma} \cdot \mathbf{E}$$

$$= \sigma_P \mathbf{E}_\perp + \sigma_H \hat{\mathbf{B}} \times \mathbf{E} + \sigma_{||} \mathbf{E}_{||}$$

$$\tilde{\sigma} = \begin{bmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{bmatrix}$$

$$\int_h \sigma_x dh = \Sigma_x$$

Ionospheric Conductivity



Fundamentals - Current Understanding/Modeling - What's next?

$$\sigma_{||} = \frac{n_e e^2}{m_e \left(\sum_i \nu_{ei} + \sum_n \nu_{en} \right)}$$

$$\tilde{\sigma} = \begin{bmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{||} \end{bmatrix}$$

$$\sigma_H = - \sum_i \left(\frac{n_i e_i^2}{m_i \nu_{in}} \right) \frac{\nu_{in} \omega_{ci}}{\nu_{in}^2 + \omega_{ci}^2} + \left(\frac{n_e e^2}{m_e \nu_{en}} \right) \frac{\nu_{en} \omega_{ce}}{\nu_{en}^2 + \omega_{ce}^2}$$

$$\sigma_P = \sum_i \left(\frac{n_i e_i^2}{m_i \nu_{in}} \right) \frac{\nu_{in}^2}{\nu_{in}^2 + \omega_{ci}^2} + \left(\frac{n_e e^2}{m_e \nu_{en}} \right) \frac{\nu_{en}^2}{\nu_{en}^2 + \omega_{ce}^2}$$

The equations are often written in different forms

Ionospheric Conductivity



Fundamentals - Current Understanding/Modeling - What's next?

$$\sigma_{||} = \frac{n_e e^2}{m_e \left(\sum_i \nu_{ei} + \sum_n \nu_{en} \right)}$$

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$\nu_{x_1 x_2} = x_1 \rightarrow x_2 \text{ collision frequency}$

$\omega_c = \frac{|e|B}{m}$

The equations are often written in different forms

What is needed to calculate conductivity?



Fundamentals - Current Understanding/Modeling - What's next?

Magnetic field

Neutral composition

Temperature

Ion/electron densities

What is needed to calculate conductivity?



Fundamentals - Current Understanding/Modeling - What's next?

To get these we need

Magnetic field

Neutral composition

Temperature

Ion/electron densities



Ion production rates

From solar photons and precipitating particles

Collisions

Plasma drift

Equilibrium densities

Ion production: Two contributions



Fundamentals - Current Understanding/Modeling - What's next?

Solar photons

Magnetospheric particles



Solar Contribution

Fundamentals - Current Understanding/Modeling - What's next?

Brekke and Moen [1993]

Author	Year	Σ_p	Σ_H
Schuster	1889, 1908	$a + b \cdot \cos \chi$	
Appleton	1937	$\text{const.} \cdot (\cos \chi)^{3/2} (v \gg \omega)$	
Metha	1978	$7.1 \cdot (\cos \chi)^{0.44}$	$13.7(\cos \chi)^{0.45}$
Senior	1980	$9.6 \cdot \cos \chi + 1.6$	$15.8 \cdot \cos \chi + 2.3$
Vickrey <i>et al.</i>	1981	$5 \cdot (\cos \chi)^{1/2}$	$10 \cdot (\cos \chi)^{1/2}$
de la Beaujardière <i>et al.</i>	1982	$10 \cdot \cos \chi + 2$	$16 \cdot \cos \chi + 3$
Robinson and Vondrak*	1984	$0.88 \sqrt{S_a} (\cos \chi)^{1/2}$	$1.5 \sqrt{S_a} (\cos \chi)^{1/2}$
Rasmussen <i>et al.</i> †	1988	$(4.5/\mathbf{B})(1 - 0.85 \cdot v^2) \cdot (1 + 0.15u + 0.05u^2)$	$(5.6/\mathbf{B}) \cdot (1 - 0.9v^2) \cdot (1 + 0.15u + 0.005u^2)$
Schlegel	1988	$6.4 \cdot (\cos(\chi - 12^\circ))^{0.54}$	
Brekke and Hall	1988	$3.05 \cos \chi + 4.06(\cos \chi)^{1/2}$	$6.24 \cos \chi + 2.85(\cos \chi)^{1/2}$
Senior	1991	$1.81 + 8.88 \cos \chi$	$21.58 - 0.21 \cdot \chi$
Moen and Brekke	1992	$S_a^{0.49} \cdot (0.34 \cos \chi + 0.93 \cdot (\cos \chi)^{1/2})$	$S_a^{0.53} (0.81 \cos \chi + 0.54(\cos \chi)^{1/2})$

* S_a being the daily 10.7 cm solar radio flux at Ottawa in units of $10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$ (adjusted to 1 A.U.).

† v is the normalized zenith angle, $v = \chi/90^\circ$, u is the normalized solar flux, $u = S_a/90$, \mathbf{B} is the magnetic field strength.

Richmond and Kamide [1998]

$$\Sigma_H(\chi, F_{10.7}) = \begin{cases} 1.8F_{10.7}^{1/2} \cos(\chi) & \text{for } \chi \leq 65, \\ \Sigma_H(65, F_{10.7}) - 0.27(\chi - 65) & \text{for } \chi > 65, \end{cases}$$

$$\Sigma_P(\chi, F_{10.7}) = \begin{cases} 0.5F_{10.7}^{2/3} \cos(\chi)^{2/3} & \text{for } \chi \leq 65, \\ \Sigma_P(65, F_{10.7}) - 0.22(\chi - 65) & \text{for } 65 < \chi \leq 100, \\ \Sigma_P(100, F_{10.7}) - 0.13(\chi - 100) & \text{for } \chi > 100, \end{cases}$$

Used in AMIE procedure



Solar Contribution

Fundamentals - Current Understanding/Modeling - What's next?

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Used in AMIE procedure

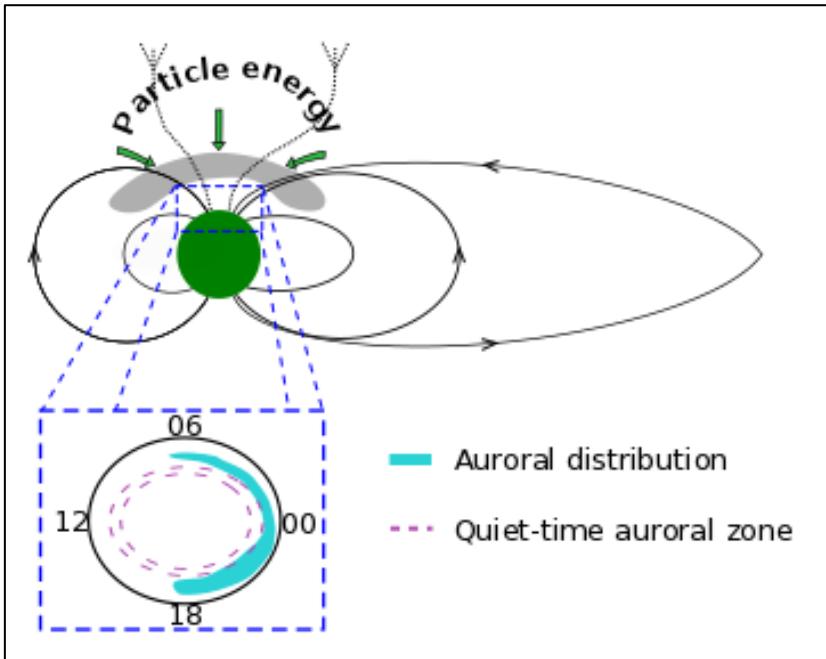
Additional considerations?

Ridley *et al.*, [2004]

Auroral Contribution

Fundamentals - Current Understanding/Modeling - What's next?

Much more uncertain component

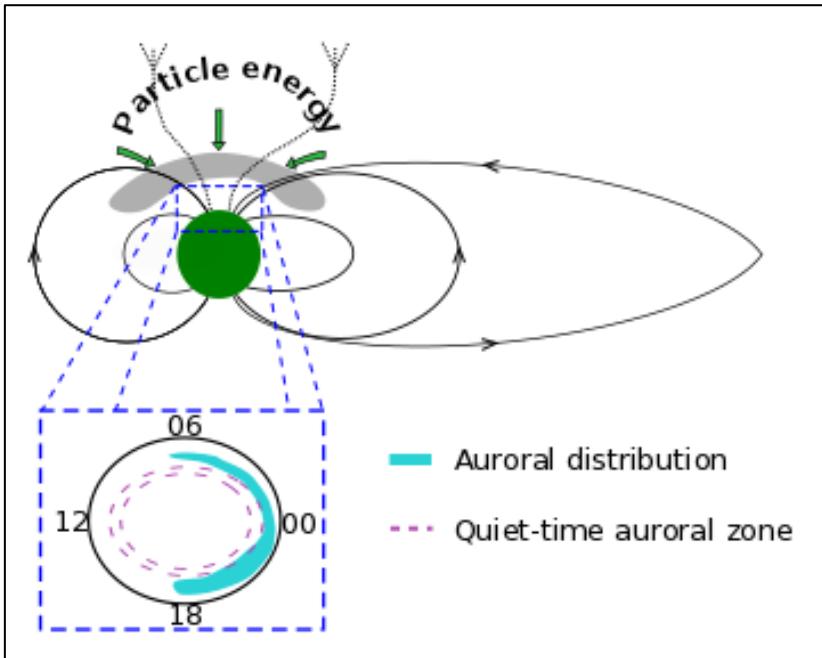


Auroral Contribution

Fundamentals - Current Understanding/Modeling - What's next?

Many approaches:

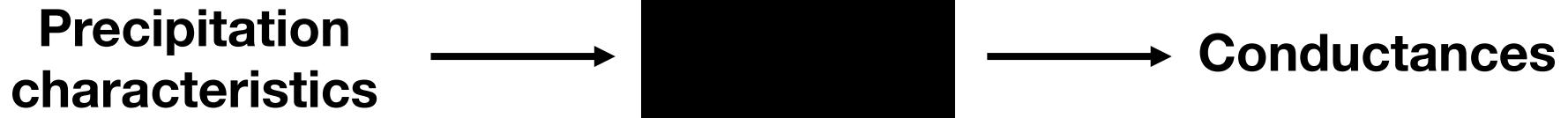
Particle data
Emission data
Magnetometer data
Ground-based
Indirect space-based
Multiple data sources at once





Auroral Contribution

Fundamentals - Current Understanding/Modeling - What's next?



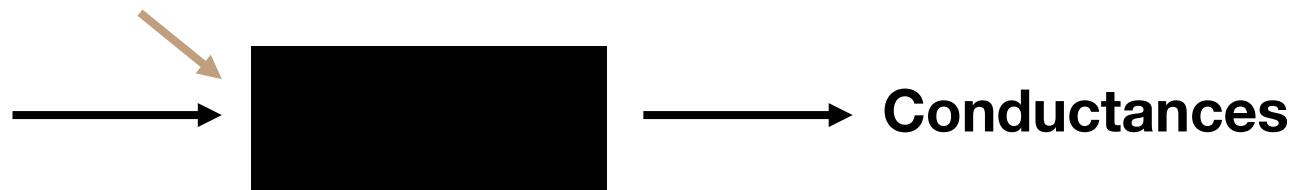


Auroral Contribution

Fundamentals - Current Understanding/Modeling - What's next?

Neutral atmosphere,
Magnetic field, Collisions

Precipitation
characteristics

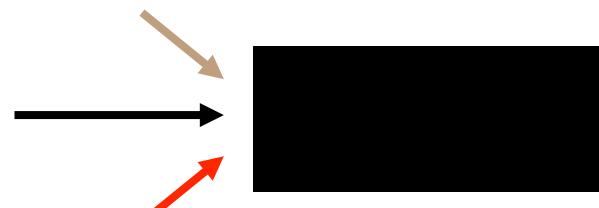


Auroral Contribution

Fundamentals - Current Understanding/Modeling - What's next?

Neutral atmosphere,
Magnetic field, Collisions

Precipitation
characteristics
(average energy &
energy flux)



Conductances

*precipitation spectrum
assumption*

1. Relationships between precipitating particles and conductances

Electrons

Spiro et al., [1982]

Robinson et al., [1987]

Protons

Hardy et al., [1989]

2. Models of conductance due to precipitating particles

Electrons

Hardy et al., [1987]

Fuller-Rowell and Evans, [1987]

Protons

Galand and Richmond, [2001]



Electrons → Conductance

Fundamentals - Current Understanding/Modeling - What's next?

Maxwellian energy particle
precipitation assumption

and

Robinson formulas

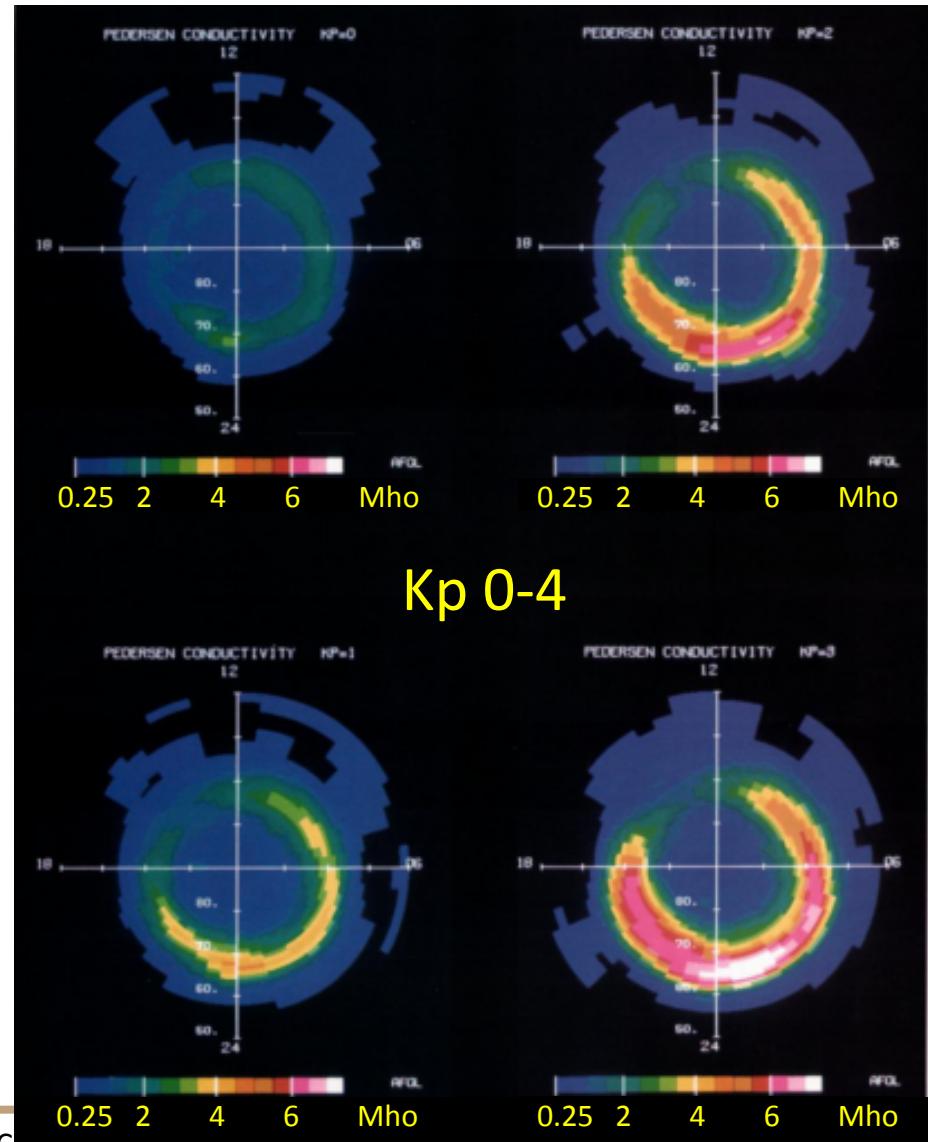
$$\Sigma_p = \frac{40E}{16 + E^2} \Phi_E^{1/2}$$
$$\frac{\Sigma_H}{\Sigma_P} = 0.45(E)^{0.85}$$

Robinson et al., [1987]

Electrons → Conductance

Fundamentals - Current Understanding/Modeling - What's next?

Height-integrated Pedersen conductances inferred from DMSP particle precipitation observations

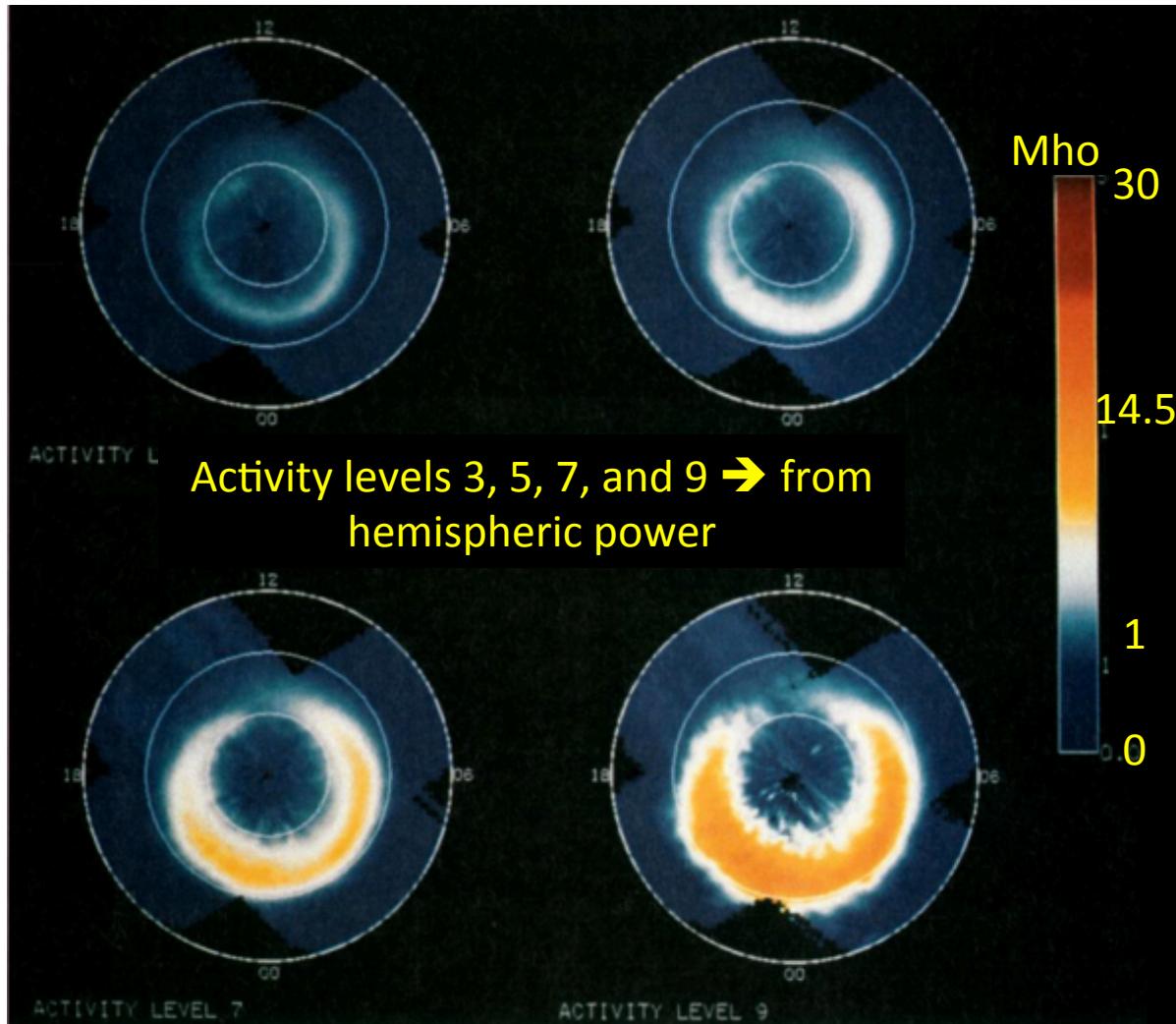


Maxwellian precipitation assumption

Hardy et al., [1987]

Electrons → Conductance

Fundamentals - Current Understanding/Modeling - What's next?



Height-integrated Pedersen conductances inferred from TIROS/NOAA particle precipitation observations

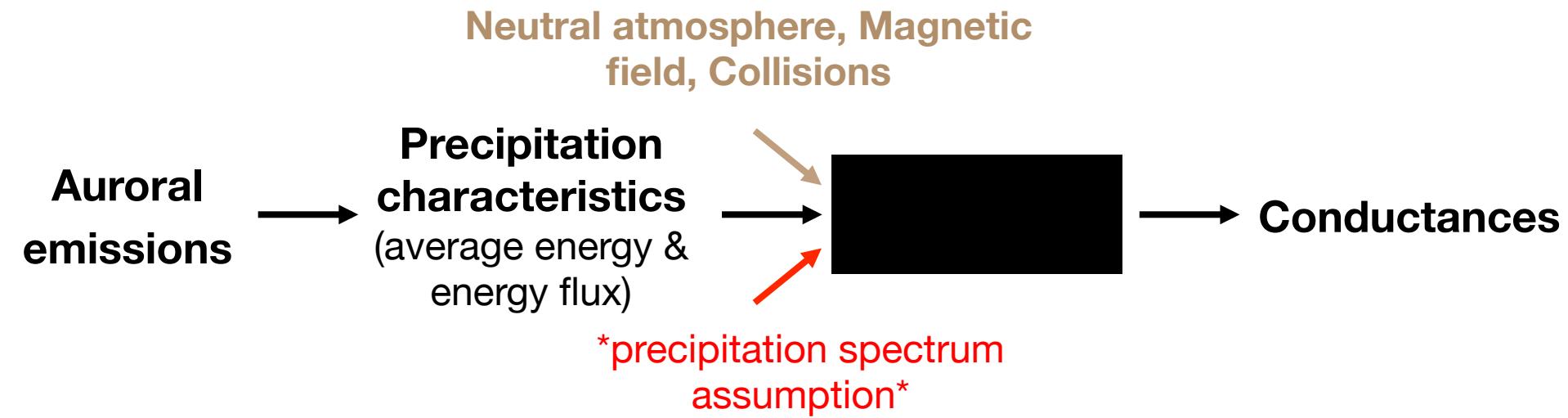
Activity levels 3, 5, 7, and 9 → from hemispheric power

Functional spectral precipitation assumption

Fuller-Rowell and Evans, [1987]

Emission data

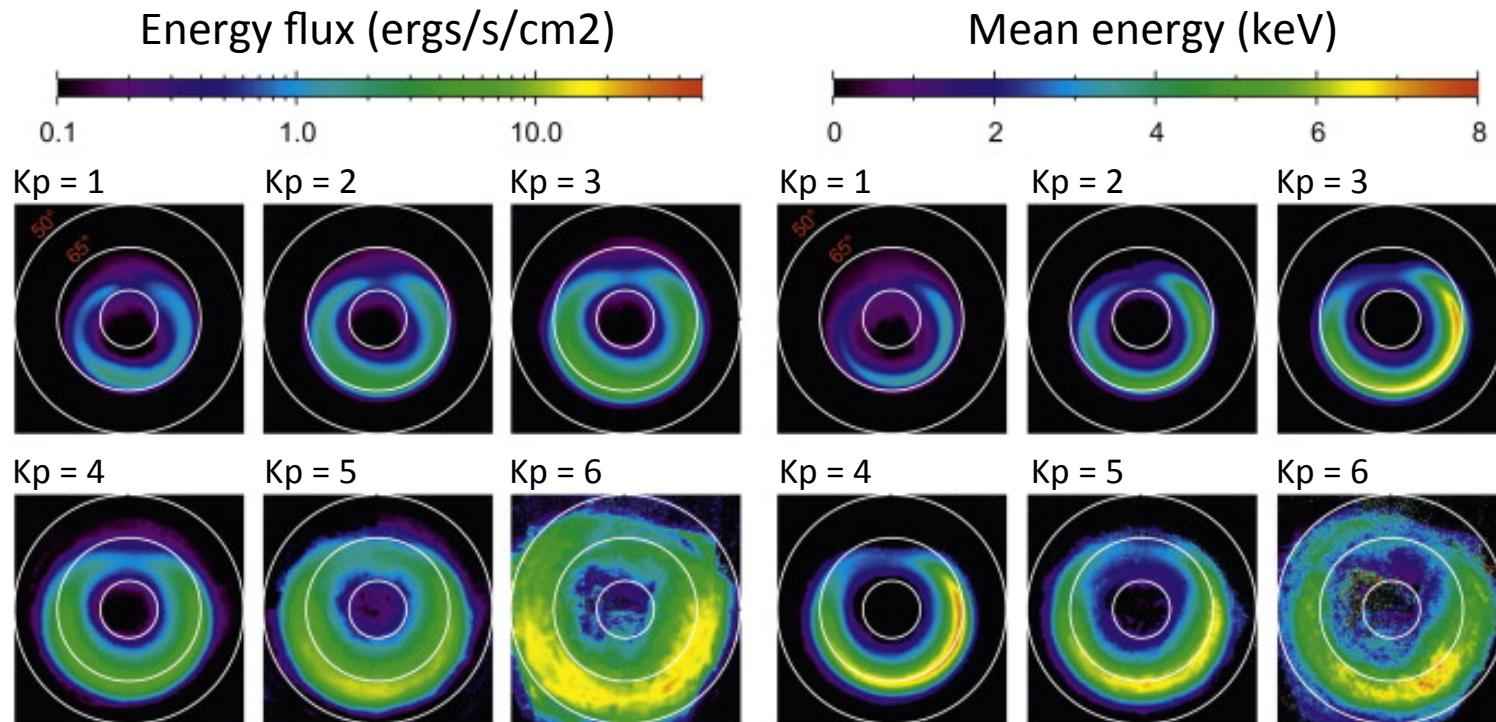
Fundamentals - Current Understanding/Modeling - What's next?



Emission data

Fundamentals - Current Understanding/Modeling - What's next?

GU VI auroral images used to create Kp-based auroral model

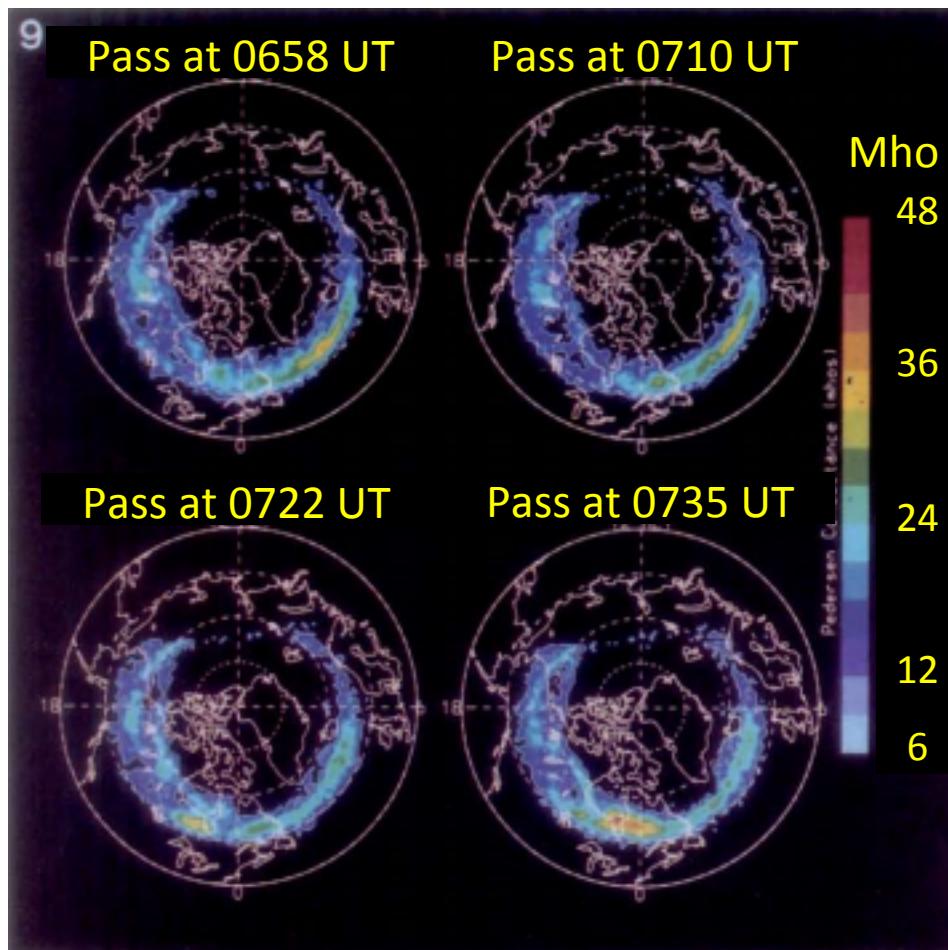


Zhang and Paxton, [2008]

Emission data

Fundamentals - Current Understanding/Modeling - What's next?

Reconstruction of conductances from combination of auroral model and emission data (consecutive DE-1 passes)



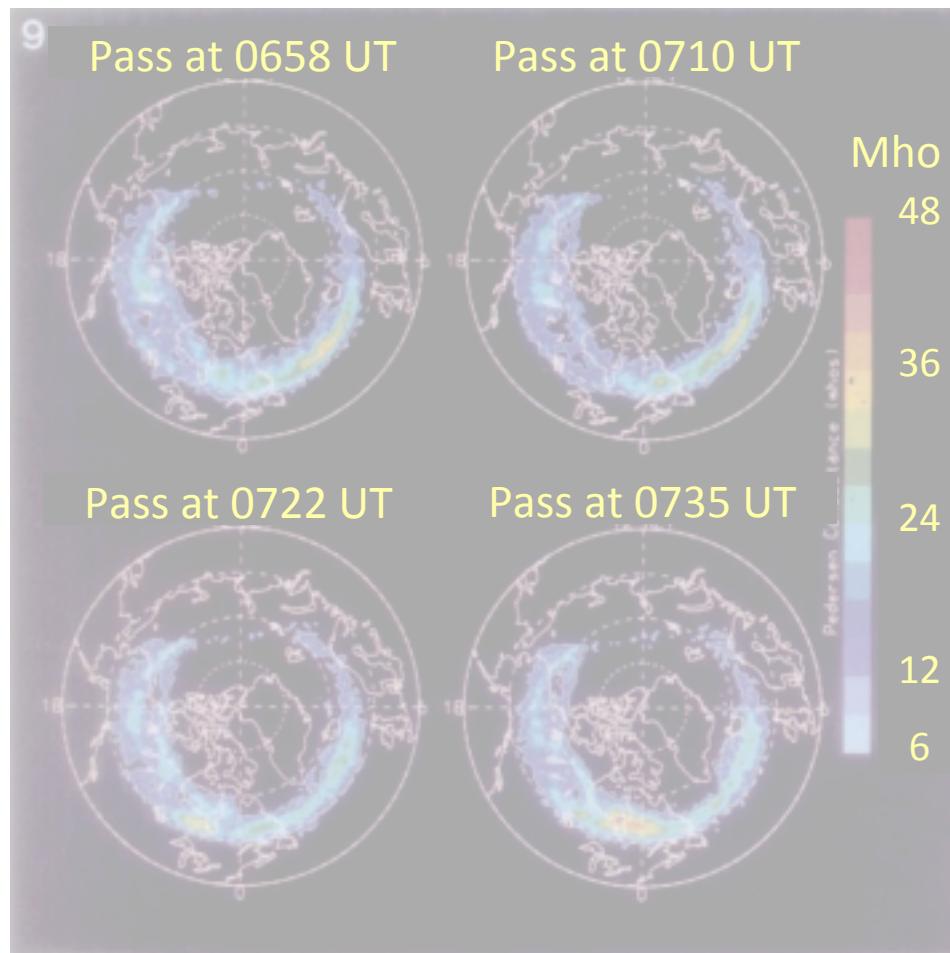
Maxwellian precipitation assumption

Lummerzheim et al., [1991]

Emission data

Fundamentals - Current Understanding/Modeling - What's next?

Reconstruction of conductances from combination of auroral model and emission data (consecutive DE-1 passes)



Maxwellian precipitation assumption

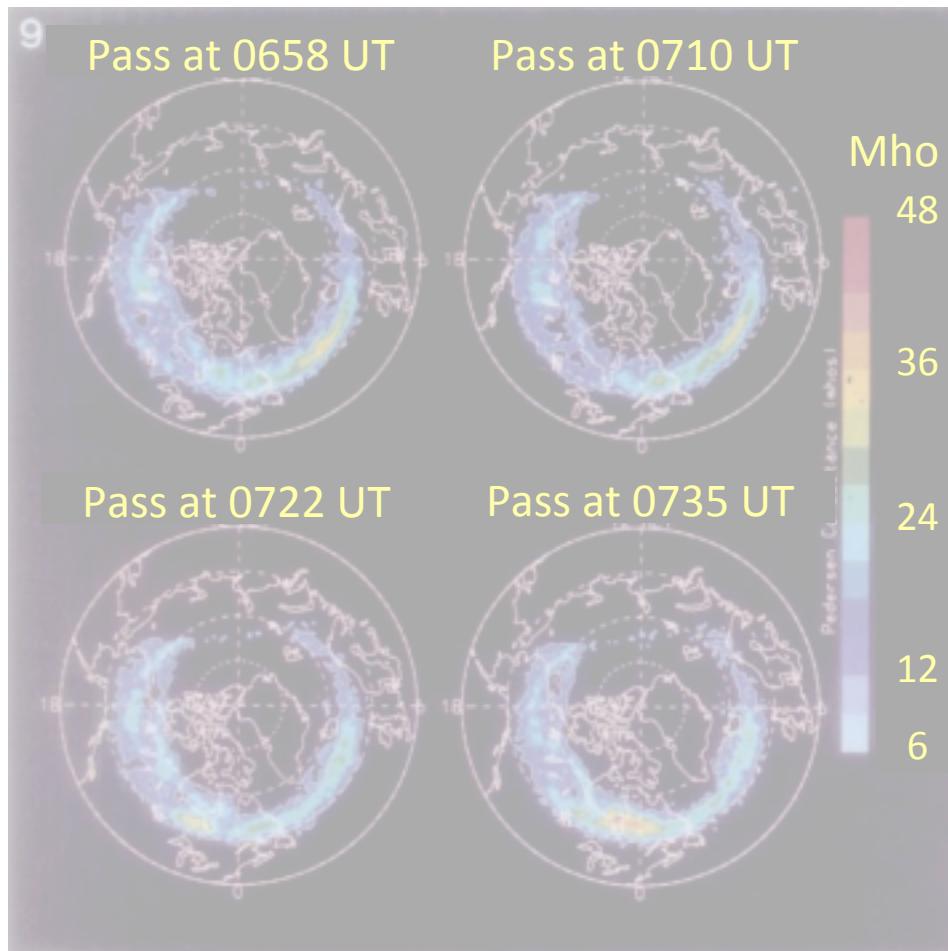
Many similar studies:

- Germany et al., [1994]
- Aksnes et al., [2002]
- Coumans et al., [2004]

Emission data

Fundamentals - Current Understanding/Modeling - What's next?

Reconstruction of conductances from combination of auroral model and emission data (consecutive DE-1 passes)



Maxwellian precipitation assumption

Many similar studies:

Germany et al., [1994]
Aksnes et al., [2002]
Coumans et al., [2004]

Substorm modeling:

Gjerloev et al., [2000]
Aksnes et al., [2002]
Coumans et al., [2004]



Magnetometer data

Fundamentals - Current Understanding/Modeling - What's next?

Ground-based

Ahn et al., [1983, 1998]

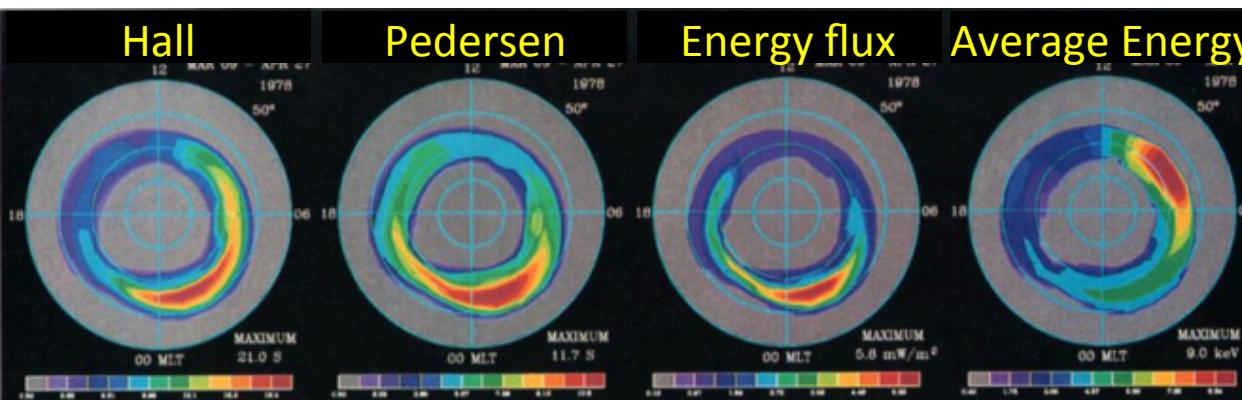
Space-based (by ensuring consistency with FACs)

Marsal et al., [2015]

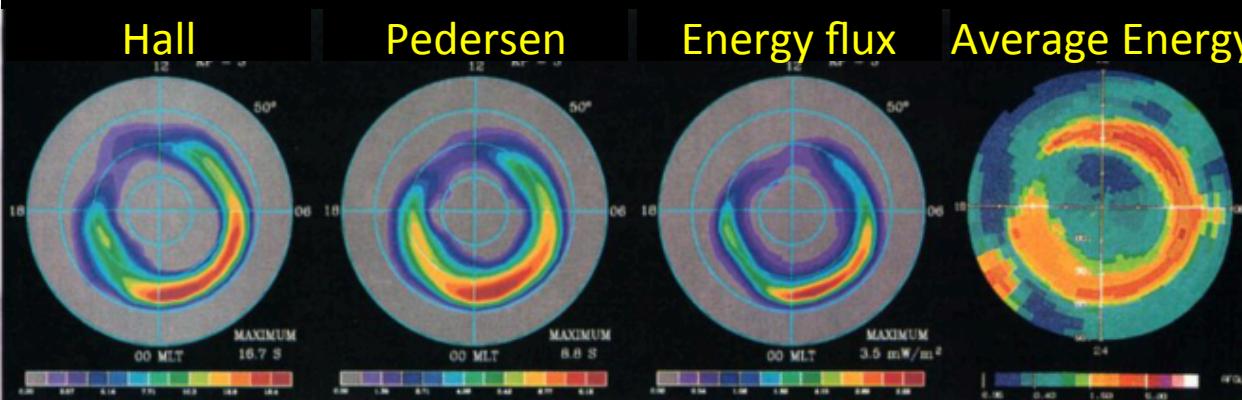
Magnetometer data: Ground-based

Fundamentals - Current Understanding/Modeling - What's next?

Empirical relationships between ground magnetometer perturbations and ISR-derived conductances developed and full distributions created



Ahn model



Hardy model

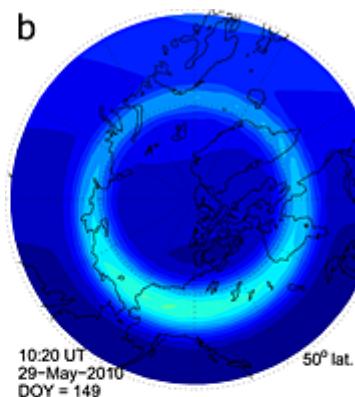
Ahn et al., [1998]

Magnetometer data: Space-based

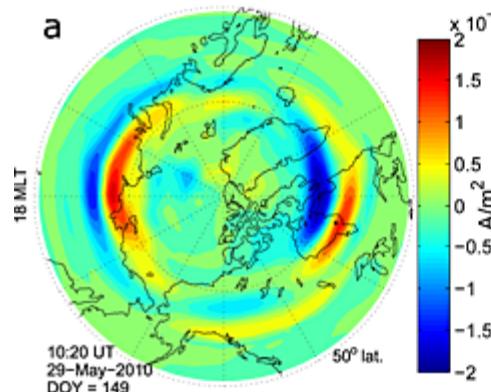
Fundamentals - Current Understanding/Modeling - What's next?

TIE-GCM
conductivities made
consistent with FACs
derived from AMPERE
magnetic perturbations
show effects of
discrete precipitation

TIE-GCM Hall
conductance

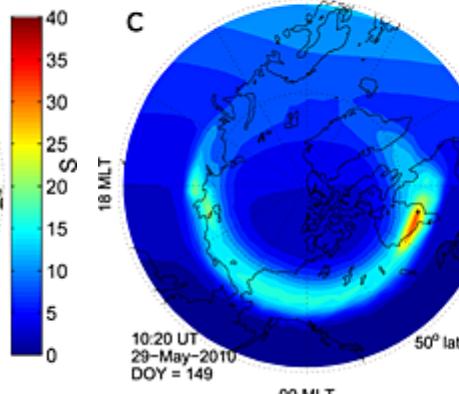


AMPERE FACs

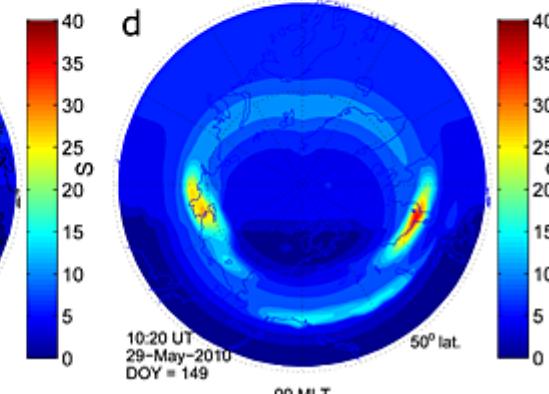


*Maxwellian precipitation
assumption*

TIE-GCM Hall
conductance consistent
with AMPERE FACs



Marklund et al., [1988]
model Hall
conductance



Multiple data sources

Fundamentals - Current Understanding/Modeling - What's next?

Data combined to estimate height-integrated conductances

AMPERE, DMSP, Oersted magnetometers
Ground magnetometers

$$\left. \begin{array}{c} \text{J}_\perp \\ \text{E}_\perp \end{array} \right\}$$

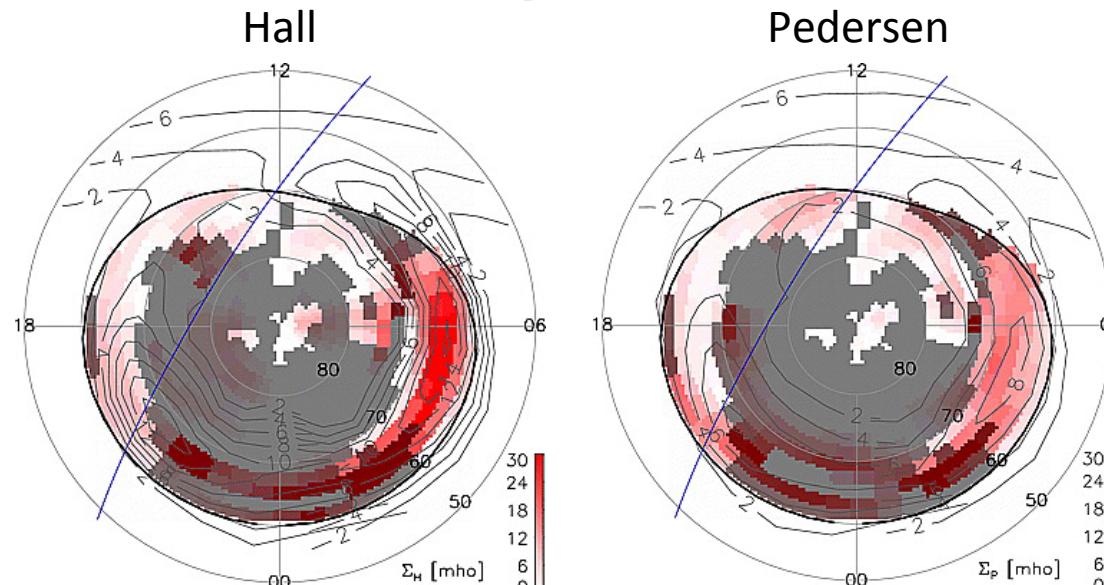
$$\Sigma_H = \frac{\hat{\mathbf{r}} \cdot (\mathbf{J}_\perp \times \mathbf{E}_\perp)}{|\mathbf{E}_\perp|^2}$$

DMSP ion drift
SuperDARN plasma convection

$$\left. \begin{array}{c} \text{J}_\perp \\ \text{E}_\perp \end{array} \right\}$$

$$\Sigma_P = \frac{\mathbf{J}_\perp \cdot \mathbf{E}_\perp}{|\mathbf{E}_\perp|^2}$$

Amm et al., [2001]



What's next?



Fundamentals - Current Understanding/Modeling - **What's next?**

What's next?



Fundamentals - Current Understanding/Modeling - **What's next?**

Past modeling:

Particle precipitation spectra assumption

2D only

Empirical modeling

Generally limited to single data source -> different approaches paint different picture

What's next?



Fundamentals - Current Understanding/Modeling - **What's next?**

Past modeling:

What is needed?

Remove Particle precipitation spectra assumption

3D ~~2D only~~

Focus on modeling small scales in global analyses ~~empirical modeling~~

Remove limitation ~~Generally limited~~ to single data source -> create
consistent ~~different approaches paint different picture~~

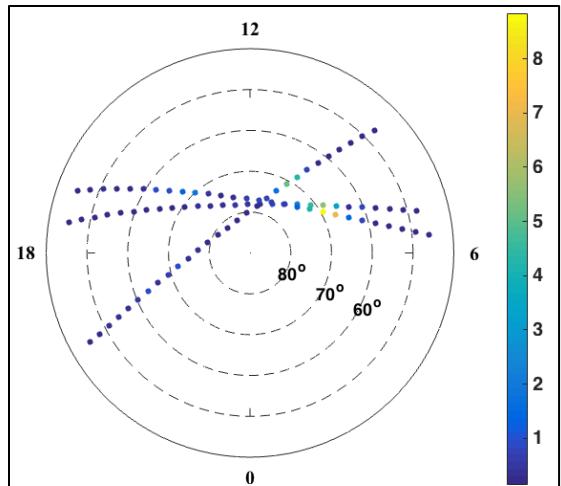
New approach: Optimal interpolation method



Fundamentals - Current Understanding/Modeling - **What's next?**

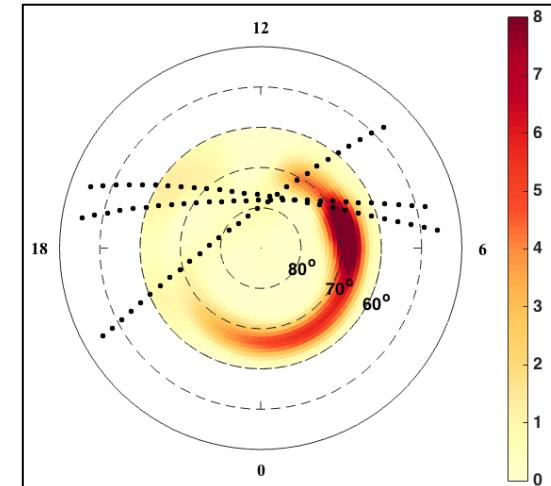
Optimally combine information from **observations** and a **background model**, taking into account **error properties** of both

Visually:



Observations

Minimize observation-model difference in least squares sense



Analysis field

McGranaghan et al., [2015a, b; 2016a, b (in. prep)]

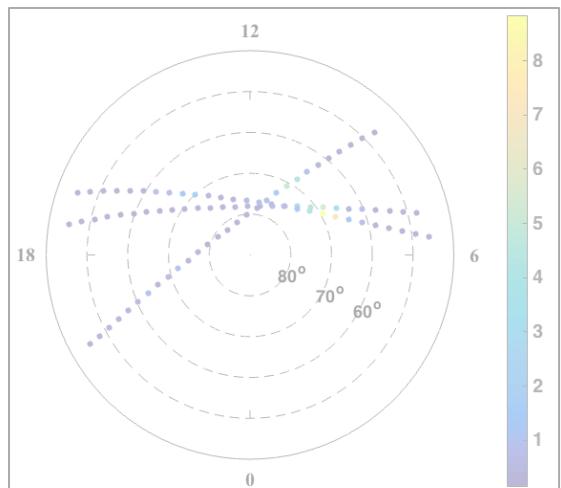
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Fundamentals - Current Understanding/Modeling - **What's next?**

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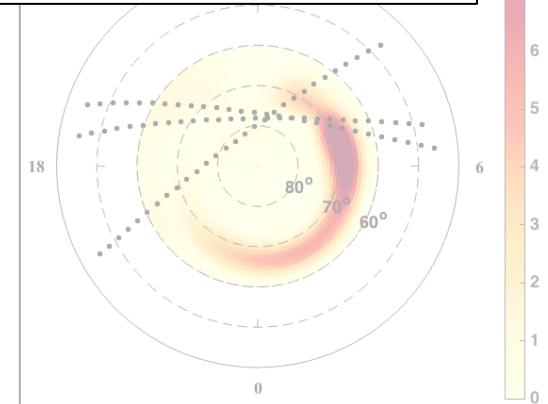
Visually:



Observations

See poster tonight:
DATA-04 Reconstruction of three-dimensional auroral ionospheric conductivities via an assimilative technique

model difference in least squares sense



Analysis field

McGranaghan et al., [2015a, b; 2016a, b (in. prep)]

Summary



Fundamentals - Current Understanding/Modeling - **What's next?**

Many different approaches to conductivity/conductance modeling

- Consistencies and disagreement

Fragmented understanding needs to be unified

New assimilative approach can bring together diverse data

- Yields estimates of uncertainty
- More difficult to use than empirical models
- How to use with magnetospheric models?



References

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Backup slides



Specialized modeling

Fundamentals - Current Understanding/Modeling - What's next?

Regional

Amm et al., [2015] – Swarm-based estimates

Auroral substorms

Germany et al., [1997]

Gjerloev et al., [2000]

Turbulent conductivities

Dimant and Oppenheim, [2011]

Better organization of conductance features

Using dynamic auroral boundary coordinates? (to my knowledge this has not been done)



Challenges

Fundamentals - Current Understanding/Modeling - **What's next?**

Issues with AMIE and MHD models likely traced to conductivity specification

Winglee et al., [1997] – AMIE

Raeder et al., [2001] – Modeling challenge study

Pulkkinen et al., [2013] – Performance of various MHD models

Time resolution of methods

Substorm dynamics require much finer resolution

Gjerloev., [2000] – emission data can produce conductances at time resolution of measurement

Three-dimensional modeling

Amm et al., [2008] – importance of 3D ionospheric specification

Multi-scale understanding

How does this impact magnetospheric modeling?



Fundamentals - Current Understanding/Modeling - **What's next?**

Where/how to place conductances?

Ridley et al., [2004]

Lotko et al., [2014]

1. Determine FACs at inner boundary ($\sim 3.5 R_E$, generally)
2. Map FACs from inner boundary to ionosphere (using background magnetic field)
3. Generate a conductance pattern

→ How?

4. Map ionospheric potential to inner boundary
5. Solve electric fields and velocity fields at inner boundary and use in MHD model

How does this impact magnetospheric modeling?



Fundamentals - Current Understanding/Modeling - **What's next?**

Where/how to place conductances?

Ridley et al., [2004]

Lotko et al., [2014]

1. Determine FACs at inner boundary ($\sim 3.5 R_E$, generally)
2. Map FACs from inner boundary to ionosphere (using background magnetic field)
3. Generate a conductance pattern
→ How?
4. Map ionospheric potential to inner boundary
5. Solve electric fields and velocity fields at inner boundary and use in MHD model

- Ways MHD codes solve conductance: Simple
- Assume uniform *Fedder and Lyon, [1987]*
 - Precipitation relationships
 - a. Diffuse = $f(\text{plasma sheet T})$
 - b. Discrete = *Knight (1972)*
 - Empirically from FACs *Ridley et al., [2004]*
 - Causal electron precipitation *Zhang et al., [2015]*
 - Introduce conductance distributions Complex



So, where are we?

Dan Weimer at *Unsolved Problems in Magnetospheric Physics* conference:

‘What is needed within the community is a comprehensive conductivity model, with software code provided. Needs to use IMF values, not just Kp. It likely would be better to use EUV indices, rather than $F_{10.7}$ ’

Other perspectives...

New approach: Optimal interpolation method



Fundamentals - Current Understanding/Modeling - **What's next?**

- Optimally combine information from **observations** and a **background model**, taking into account **error properties** of both

$$\vec{x}_a = \vec{x}_b + K (\vec{y} - H \vec{x}_b)$$

$$K = P_b H^T (H P_b H^T + R)^{-1}$$

\vec{x}_a — Analysis field

\vec{x}_b — Background model

K — Kalman gain

\vec{y} — Observations

H — Forward operator

P_b — Background model error covariance

R — Observational error covariance

New approach: Assimilation



Fundamentals - Current Understanding/Modeling - **What's next?**

Objective analysis and empirical orthogonal functions allow:

- Estimates of conductivity in 3D, without assumption of particle precipitation spectrum
- Facilitate bringing together growing, diverse data
- Provides estimate of uncertainty in solution
- More complex than empirical models
- Not capable of forecasting, currently
- How to bring into magnetospheric models?

Conductance control of magnetosphere

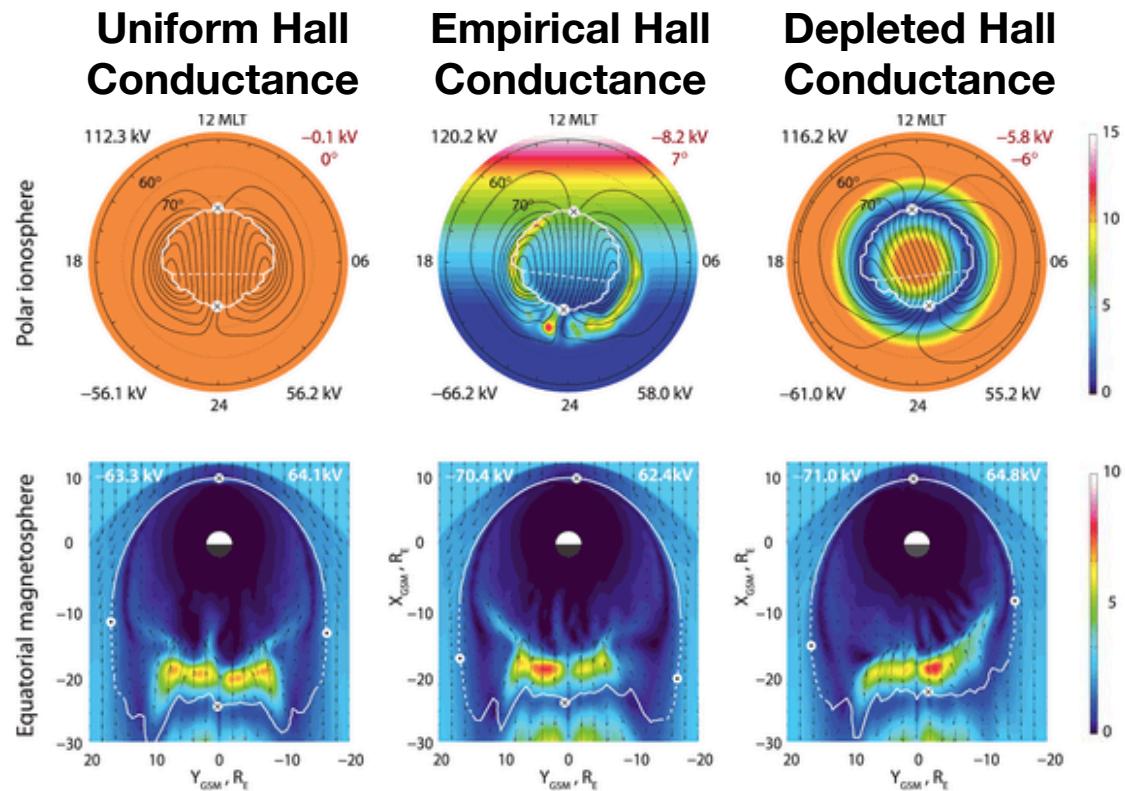


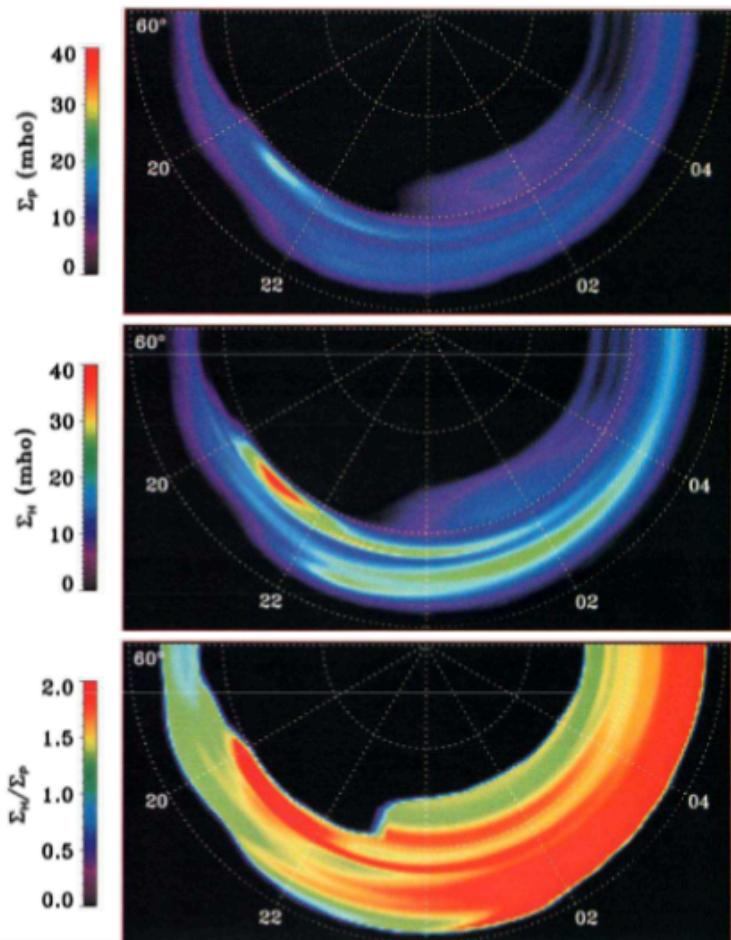
Fig. 3. Effect of ionospheric Hall conduction on convection in the ionosphere and plasmasheet. Top: Simulated ionospheric convection (8 kV contours) and Hall conductance distributions (color). The distribution of Pedersen conductance is a uniform 5 S for the simulations in the left and right panels and is similar to the Hall distribution in the causal conductance case but smaller in magnitude. Total cross polar cap potential is given in the upper left of each panel. The dusk minimum and dawn maximum are given below each panel. The white contour is the x-line mapped

along magnetic field lines from the magnetosphere. The circled cross locates its intersection with the zero-potential contour. Bottom: Simulated equatorial magnetosphere with velocity vectors overlaid on velocity magnitude in color. $B_z = 0$ contour (magnetic x-line) is shown in white. Minimum and maximum reconnection potentials from Fig. 4 occur at circled dots, with location uncertainty confined to the dashed segments where the reconnection rate is nearly zero. The circled cross is the field-line mapping of the corresponding point in the ionosphere.

Lotko et al. [2014]

Electrons → Substorm Conductance

Fundamentals - Current Understanding/Modeling - What's next?



Text here

*Functional spectral precipitation assumption?

Plate 1. Polar plots showing the Pedersen conductance, the Hall conductance, and the Hall to Pedersen ratio as function of invariant latitude and a generalized magnetic local time derived from the average passes shown in Figure 1.

Gjerloev and Hoffman, [2000]



Combination of auroral and solar contributions

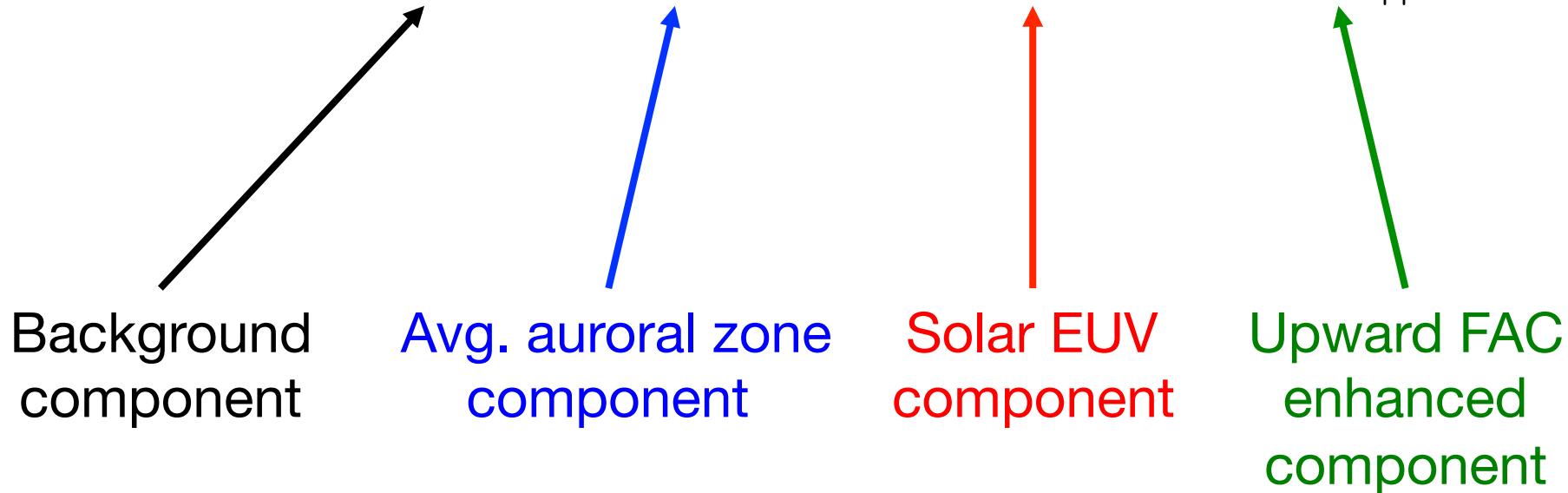
$$\sigma_{\text{total}}^2 = \sigma_{\text{auroral}}^2 + \sigma_{\text{solar}}^2 \quad \text{Correct}$$

↑
↓
These operations do not commute

$$\Sigma_{\text{total}}^2 = \Sigma_{\text{auroral}}^2 + \Sigma_{\text{solar}}^2 \quad \text{Introduces error}$$

$$\Sigma_{P_{\text{total}}}^2 = \Sigma_{P_0}^2 + \Sigma_{P_g}^2 + \Sigma_{P_{UV}}^2 + \Sigma_{P_{j||}}^2$$

$$\Sigma_{H_{\text{total}}}^2 = \Sigma_{H_0}^2 + \Sigma_{H_g}^2 + \Sigma_{H_{UV}}^2 + \Sigma_{H_{j||}}^2$$



Multiple data sources

Fundamentals - Current Understanding/Modeling - What's next?

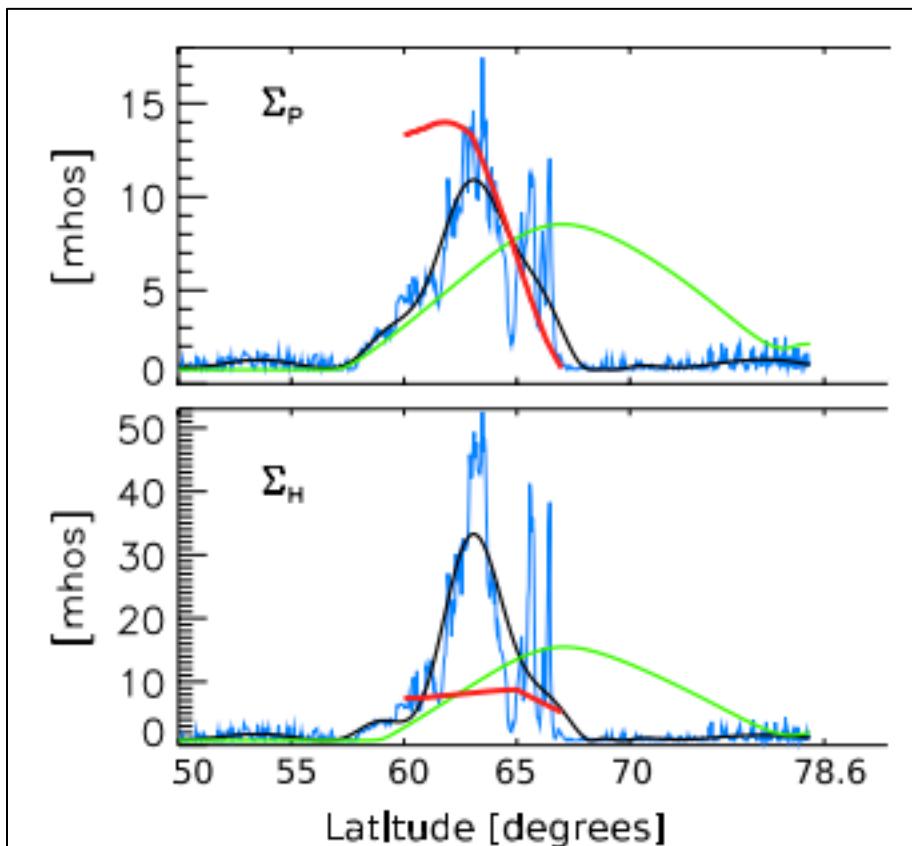


Figure 13. Conductance data along the F15 DMSP track (2100 to 1100 MLT, cf., Figures 9 and 10; 0251:0422 UT). Σ^{DMSP} (raw: blue; filtered: black), conductance model (green) and calculated from electromagnetic data (red).

Green et al., [2007]

$$\Sigma_H = \frac{\hat{\mathbf{r}} \cdot (\mathbf{J}_\perp \times \mathbf{E}_\perp)}{|\mathbf{E}_\perp|^2}$$

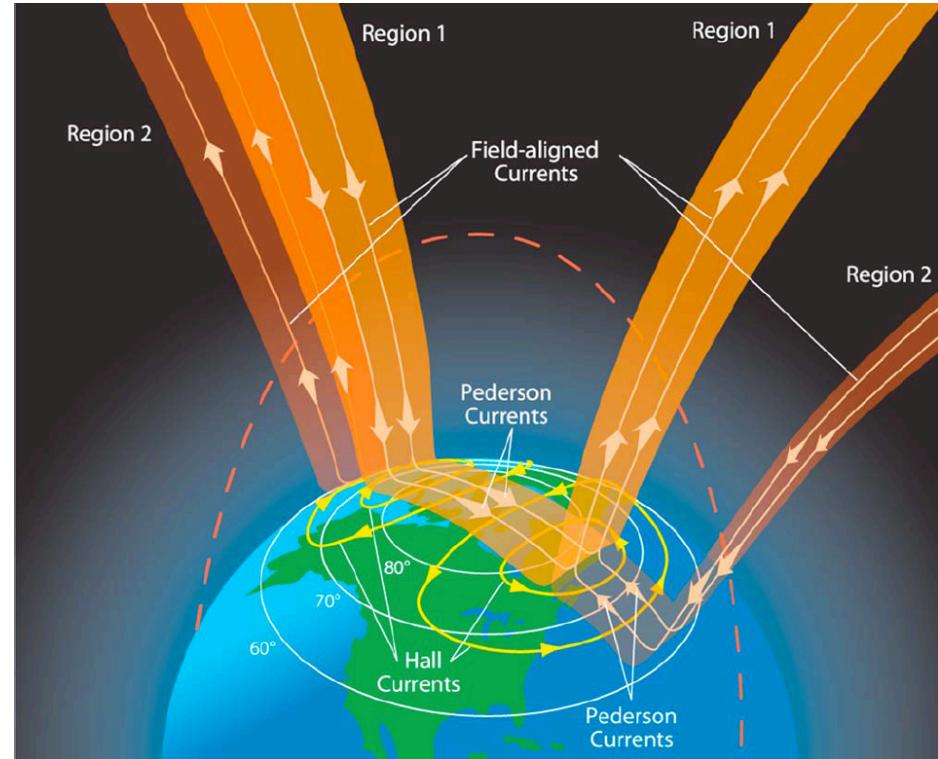
$$\Sigma_P = \frac{\mathbf{J}_\perp \cdot \mathbf{E}_\perp}{|\mathbf{E}_\perp|^2}$$

Amm et al., [2001]

Ionospheric Conductivity



Fundamentals - Current Understanding/Modeling - What's next?



$$\mathbf{J} = \tilde{\sigma} \cdot \mathbf{E}$$

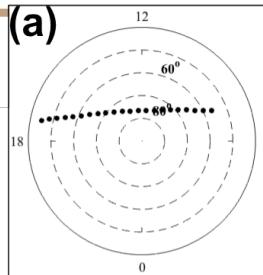
$$= \sigma_P \mathbf{E}_\perp + \sigma_H \hat{\mathbf{B}} \times \mathbf{E} + \sigma_{\parallel} \mathbf{E}_{\parallel}$$

$$\tilde{\sigma} = \begin{bmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{\parallel} \end{bmatrix}$$

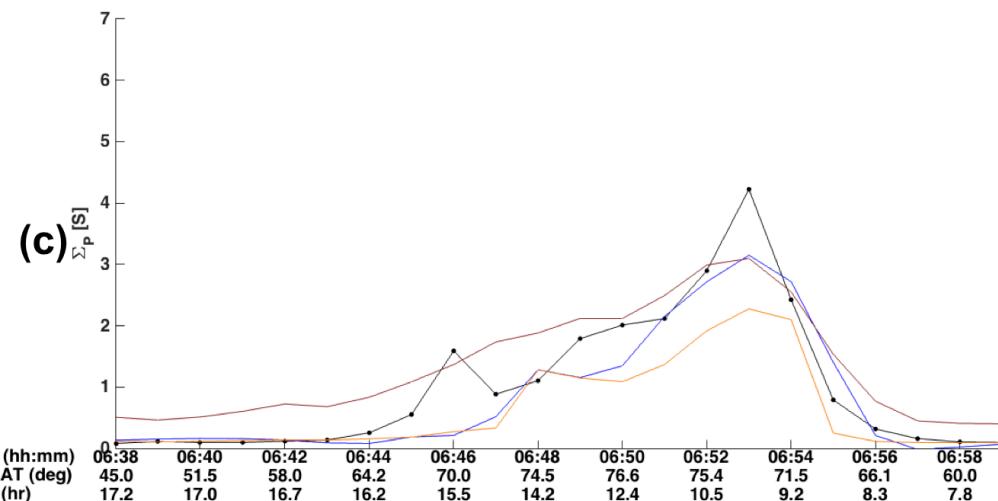
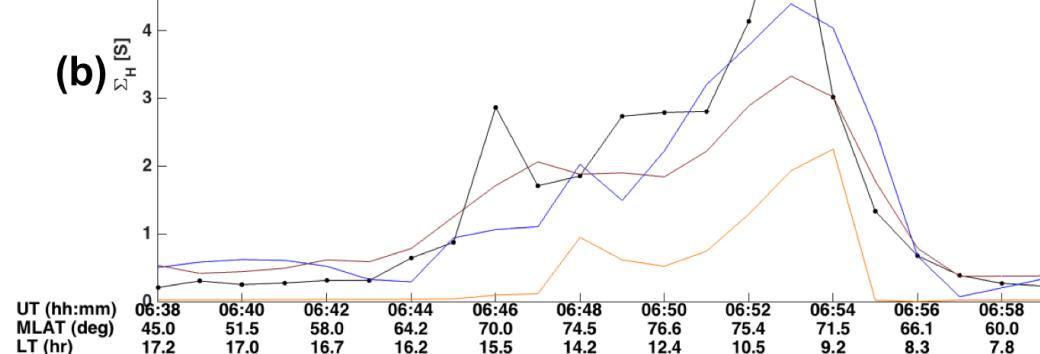
$$\int_h \sigma_x dh = \Sigma_x$$

Multiple data sources

Fundamentals - Current Understanding/Modeling - What's next?



- DMSP
- OI
- Robinson et al., [1987]
- Fuller-Rowell and Evans, [1987]



McGranaghan et al., [2016]

New approach: Optimal interpolation method



Fundamentals - Current Understanding/Modeling - **What's next?**

- Optimally combine information from **observations** and a **background model**, taking into account **error properties** of both
- **Background model**: EOF-based mean
- **Observations**: DMSP particle precipitation data
- **Error properties**:
 - For background model: Estimated from EOFs
 - For DMSP particle precipitation data: Poisson statistics for individual spectra

\vec{x}_a — Analysis field

\vec{x}_b — Background model

K — Kalman gain

\vec{y} — Observations

H — Forward operator

P_b — Background model error covariance

R — Observational error covariance

$$\vec{x}_a = \vec{x}_b + K (\vec{y} - H \vec{x}_b)$$

$$K = P_b H^T (H P_b H^T + R)^{-1}$$



Obs Creation: ¹Particle Spectrum - ²Conductivity - ³Integration - ⁴Accumulate

Conductance observation creation:

1. DMSP in-situ electron energy spectrum
2. Conductivity profiles (from GLOW model)
3. Integrate over 80-200 km and apply to all spectra for satellite pass
4. Accumulate over analysis window

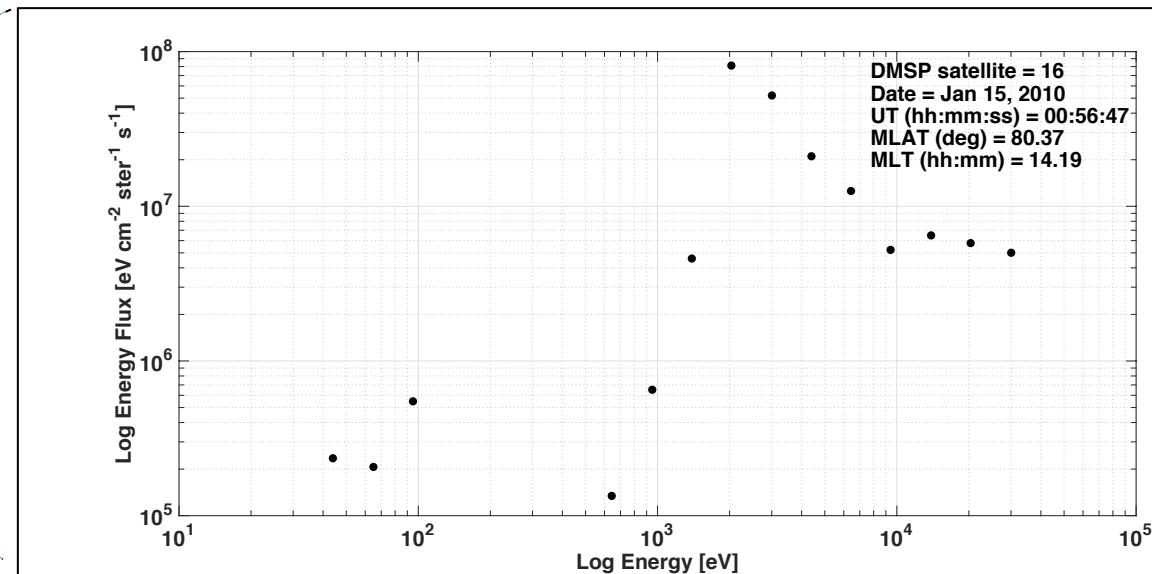
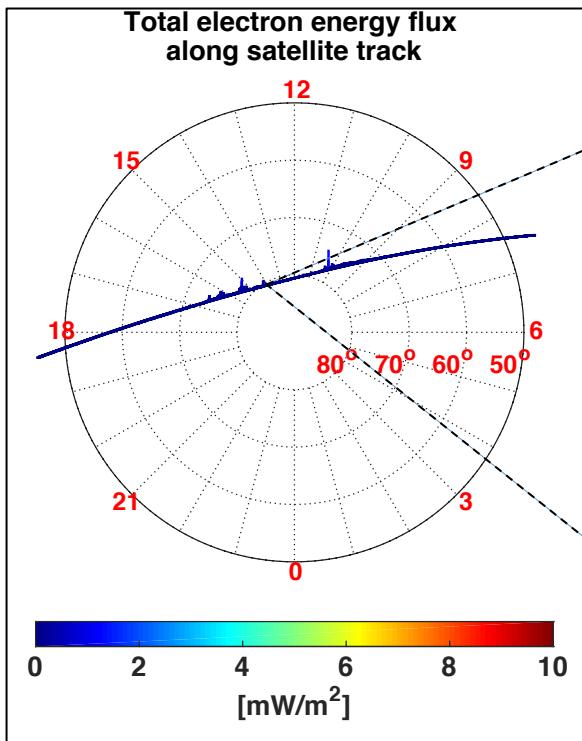


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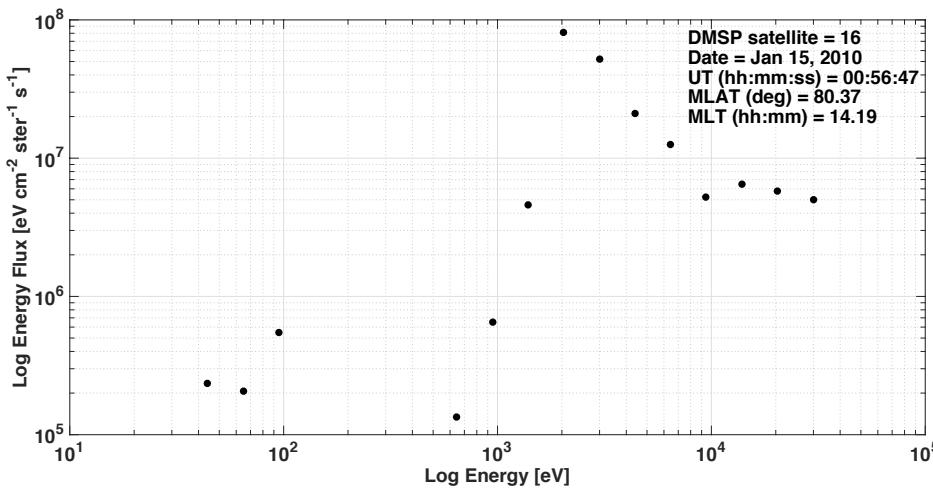


Obs Creation: ¹Particle Spectrum - ²Conductivity - ³Integration - ⁴Accumulate

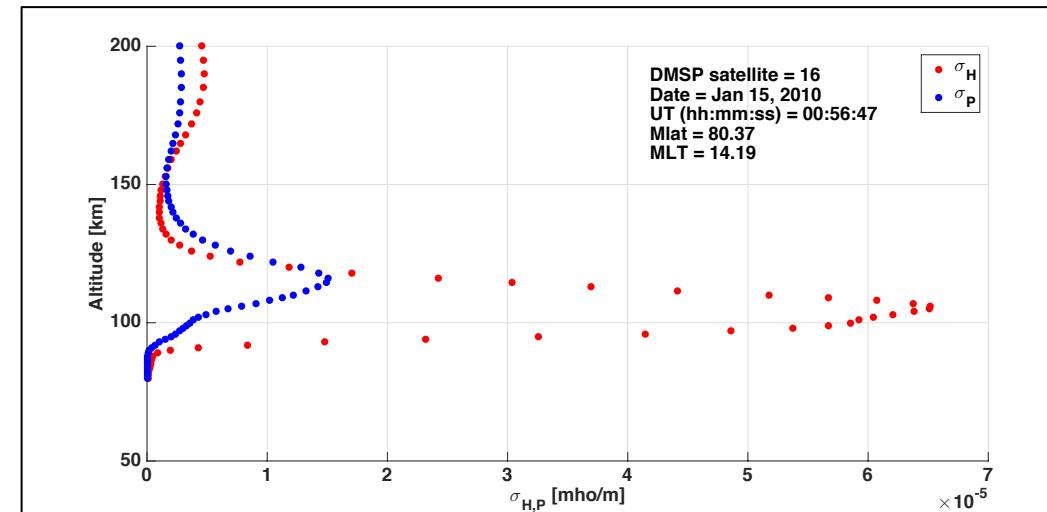
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3. Integrate over 80-200 km and apply to all spectra for satellite pass
4. Accumulate over analysis window

Obs Creation: ¹Particle Spectrum - ²Conductivity - ³Integration - ⁴Accumulate



GLOW +
conductivity



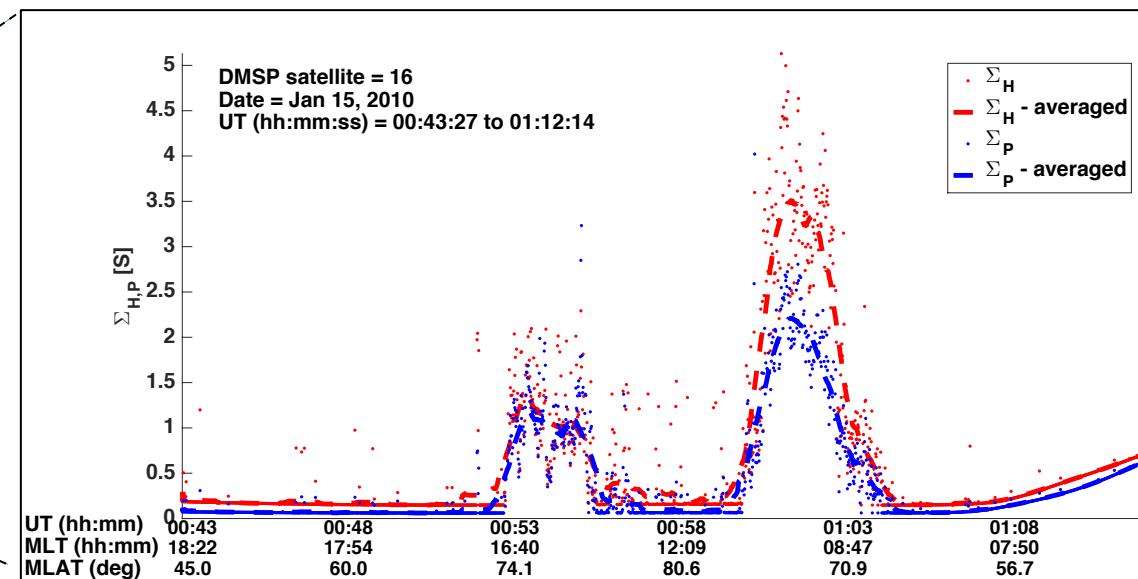
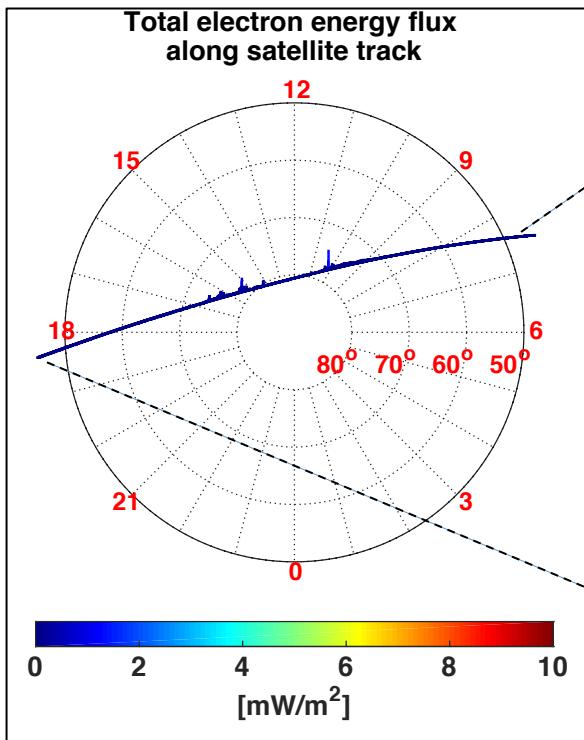


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Obs Creation: ¹Particle Spectrum - ²Conductivity - ³Integration - ⁴Accumulate





Obs Creation: ¹Particle Spectrum - ²Conductivity - ³Integration - ⁴Accumulate

Conductance observation creation:

1. DMSP in-situ electron energy spectrum
2. Conductivity profiles (from GLOW model)
3. Integrate over 80-200 km and apply to all spectra for satellite pass
4. Accumulate over analysis window

Obs Creation: ¹Particle Spectrum - ²Conductivity - ³Integration - ⁴Accumulate

Jan. 15, 2010

UT time (hh:mm:ss) =
00:00:00 to 01:00:00

4. Accumulate over analysis window

